For convenience, ω_0' in (8) is given in tabular form in Table 1 for several values of r and n.

TABLES OF PROTOTYPE ELEMENT VALUES[4]

Tables 2 to 6 (p. 694) give element values for prototype maximally flat impedance-transforming networks for $n=2$, 4, 6, 8, and 10 reactive elements. After the designer has arrived at values for r, $\mathcal{W}$, and n, the normalized element values can be obtained from the tables. Since the networks presented in Tables 2 through 6 are antimetric [3], i.e., half of the network is the inverse of the other half, only half of the network element values need be presented; the remaining elements can be computed from single equations [2]. However, for the convenience of the reader, all element values of the networks are presented in Tables 2 through 6.[5]

EXAMPLE

A numerical example will serve to demonstrate the use of Tables 2 through 6 and Fig. 3, and Table 1. Suppose that a designer desires a maximally flat impedance-transforming network for an $r=20$ impedance ratio, over the band from 500 to 1000 Mc/s. The required fractional bandwidth is given by

$$w = \frac{f_b - f_a}{f_m} = \frac{2(f_b - f_a)}{f_b + f_a} \qquad (10)$$

which for this example gives

$$w = \frac{2(1000 - 500)}{1000 + 500} = 0.667.$$

From Fig. 3, it is found that this value of fractional bandwidth and impedance ratio lies between the $n=2$ and $n=4$ curves, so that $n=4$ reactive elements are necessary. (Two reactive elements would give a fractional bandwidth of only 0.5.) This will give an operating bandwidth somewhat larger than is actually required ($w=0.79$), which is often desirable.

Next, from Table 3, for $n=4$ reactive elements, the element values

$$g_1 = 2.56209$$
$$g_2 = 0.64144$$
$$g_3 = 12.82873$$
$$g_4 = 0.12810$$

are obtained; and from Table 1 [or (9)] ω_0' is found to be 0.77012. The computed transmission response of the network is graphed in Fig. 4.

SCALING OF THE NORMALIZED DESIGN

After a designer has selected a normalized design, the element values required for a specific application are easily determined by scaling. Let R be the desired resistance level of one of the terminations, while R' is the corresponding resistance of the normalized design. Similarly, let ω_b be the radian frequency of the upper 3-dB frequency of the desired operating band, while $\omega_b'=1$ is the corresponding frequency for the normalized design. Then the scaled element values are computed using

$$R_k = R_k'\left(\frac{R}{R'}\right) \qquad (11)$$

$$C_k = C_k'\left(\frac{\omega_b'}{\omega_b}\right)\frac{R'}{R} \qquad (12)$$

$$L_k = L_k'\left(\frac{\omega_b'}{\omega_b}\right)\frac{R}{R'} \qquad (13)$$

where R_k', C_k', and L_k' are for the normalized design and R_k, C_k, and L_k are for the scaled design.

ACKNOWLEDGMENT

The computer program for the compilation of the tables of element values was written by W. Wiebenson and Miss E. Tessman.

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REFERENCES

[1] G. Szentirmai, "Band-pass matching filter in the form of polynomial low-pass filter," *IEEE Trans on Circuit Theory (Correspondence)*, vol. CT-11, pp. 177–178, March 1964.
[2] G. L. Matthaei, "Tables of Chebyshev impedance transforming networks of low-pass filter form", *Proc. IEEE*, vol. 52, pp. 939–963, August 1964.
[3] E. A. Guillemin, *Synthesis of Passive Networks*, ch. 11. New York: Wiley, 1957.
[4] E. G. Cristal, L. A. Robinson, B. M. Schiffman, and L. Young, "Novel microwave filter design techniques," Seventh Quarterly Progress Rept., Section IV, SRI Project 4344, Contract DA 36-039-AMC-00084(E), Stanford Research Inst., Menlo Park, Calif., October 1964.

[4] The derivation of the tables is given in Cristal, et al. [4].

[5] The element values were obtained by a continued fraction expansion of the input impedance of the network. Because of a loss of significant digits in the continued fraction expansion, the element values for second half of the network as given in the tables may be in error in the fourth decimal place. In those cases where the error is significant the element values of the second half of the network should be obtained from the element values of the first half of the network by the relationships given in Matthaei [2].

Tables of Stub Admittances for Maximally Flat Filters Using Shorted Quarter-Wave Stubs

Consider a symmetrical filter consisting of lossless shorted quarter-wave stubs spaced a quarter wavelength apart on a uniform, lossless line. The tables given here list the normalized characteristic stub admittances k_r necessary for a maximally flat response.

The insertion loss, when the filter is inserted between a generator and a load, both of which have real admittances equal to the characteristic admittance of the transmission line, is given by the relation

$$\frac{P_0}{P_L} = 1 + K_n \frac{\cos^{2n}\theta}{\sin^2\theta} \qquad (1)$$

where n is the number of shorted stubs of length l,

$$\theta = 2\pi l/\lambda \qquad (2)$$

$$K_n = \left(\frac{k_1(k_2+2)\cdots(k_n+2)}{2}\right)^2 \qquad (3)$$

Manuscript received June 25, 1965. This paper was presented in part at the IEEE G-MTT Symposium, May 20, 1963.

and where $k_r = Y_{0r}/Y_0$ is the normalized characteristic admittance of the rth stub.

For example, in a two-stub filter, $n=2$, and symmetry demands that the characteristic impedances of the two stubs be equal, hence

$$\frac{1}{k_1} = \frac{1}{k_2}.$$

The insertion loss is given by (1),

$$\frac{P_0}{P_L} = 1 + \frac{[k_1(k_1+2)]^2\cos^4\theta}{4\sin^2\theta}$$

$$K_2 = [k_1(k_1+2)]^2/4.$$

The following tables give $10 \log K_n$, and the required normalized characteristic admittances of the stubs for various practical values up to ten stubs. Since the filters are symmetrical, only the values for the first half of the filter are tabulated.

Three-Stub Filter

$10 \log K_3$	k_1	k_2
−12.728	0.100	0 200
− 0.944	0.300	0.600
+ 5.46	0.500	1.000
+10.138	0.700	1 400
+15.56	1.000	2 000
+21.156	1.400	2.800
+27.604	2.000	4.000
+31.904	2.5	5.0
+35.563	3.0	6 0

Four-Stub Filter

$10 \log K_4$	k_1	k_2
− 5.17	0.1	0.292
+ 3.253	0.2	0.571
+13.329	0.4	1.109
+25.668	0.8	2.141
+35.909	1.3	3.395
+44.873	1 9	4.877
+56.734	3 0	7.568

Five-Stub Filter

$10 \log K_5$	k_1	k_2	k_3
+ 3.452	0.100	0.366	0.532
13.577	0.200	0.694	0.989
20.523	0.300	1.005	1.410
26.002	0.400	1.304	1.808
30.601	0.500	1.596	2.193
38.16	0.700	2.166	2.933
44.324	0.900	2.724	3.648
54.172	1.300	3.819	5.038
66.970	2.000	5 702	7.403
77.874	2.800	7.829	10.058

Six-Stub Filter

$10 \log K_6$	k_1	k_2	k_3
+13.378	0.100	0.419	0 755
25.469	0.200	0.774	1.329
33.805	0.300	1.105	1 838
40.388	0.400	1.422	2.314
50.721	0.600	2.031	3.207
58.863	0.800	2.622	4.055
65.668	1.0	3.202	4.878
76.755	1.4	4.343	6.487
85.687	1 8	5.468	8.045
96 571	2.4	5 141	10.359

Seven-Stub Filter

$10 \log_{10} K_7$	k_1	k_2	k_3	k_4
24.63	0 1	0.4556	0 9269	1 1425
38.78	0 2	0.8259	1 5687	1 8856
56.24	0 4	1.4949	2.6514	3 1129
77.83	0.8	2.7308	4.5506	5.2396
85.77	1.0	3.3269	5.4458	6.2379
104.521	1.6	5.0774	9.0398	9.1249
125.521	2 6	7.9395	12.2306	13.7822

Eight-Stub Filter

$10 \log_{10} K_8$	k_1	k_2	k_3	k_4
37.11	0.1	0.480	1.050	1.455
53.35	0.2	0.860	1.735	2.305
73.35	0.4	1.543	2.881	3.691
98.06	0.8	2.802	4.885	6.080
107.14	1.0	3.409	5.829	7.199
128.19	1.6	5.189	8.562	10.433
139.08	2.0	6.359	10.340	12.535

Nine-Stub Filter

$10 \log K_9$	k_1	k_2	k_3	k_4	k_5
50.68	0.1	0.4974	1.138	1.689	1.898
69.02	0.2	0.8838	1.852	2.613	2.890
91.56	0.4	1.577	3.043	4.112	4.491
119.39	0.8	2.851	5.121	6.688	7.237
129.6	1.0	3.456	6.098	7.894	8.251
153.3	1.6	5.266	8.928	11.377	12.231
176.0	2.4	7.624	12.592	15.879	17.022

Ten-Stub Filter

$10 \log K_{10}$	k_1	k_2	k_3	k_4	k_5
65.24	0.1	0.510	1.203	1.866	2.245
85.66	0.2	0.901	1.938	2.842	3.340
110.753	0.4	1.601	3.161	4.423	5.102
141.69	0.8	2.887	5.292	7.138	8.117
153.07	1.0	3.506	6.294	8.408	9.257
179.39	1.6	5.322	9.195	12.076	13.596
204.651	2.4	7.699	12.949	16.815	18.851

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A Simple Design Procedure for Small Percentage Bandwidth Round-Rod Interdigital Filters

The continued correspondence concerning interdigital filter [1] prompted me to submit this correspondence. The concepts and design procedures to be described have been found to be practically useful for determining the geometry of the coupled-rod structure, and because these procedures are apparently much simpler than those presently available, publication of the subject matter may be helpful to others. It will be apparent that some of this material is tutorial in nature—it is included to clarify the other information being presented.

I. CONCERNING THE OVERALL DEVICE

However else the device may be described, when used to produce small percentage, i.e., less than 10 per cent, pass bands, the first-order phenomenon involved in a so-called interdigital filter is identical to that occurring in direct coupled-resonator filters with groundings arranged so that the magnetic field coupling is in phase with the electric field coupling; i.e., the so-called "capacity aiding" connection in the $n=2$ IF interstages of old. Thus, the concepts and quantitative procedures described in [2] are directly applicable to the design, adjustment, and alignment of these filters. Specifically the device is adequately designed if, with all other resonances short-circuited, one correctly adjusts the coefficients-of-couplings between adjacent resonances; the singly loaded Q of the input and output resonances; and the resonant frequency of each resonance.

If the foregoing is true, the following helpful questions arise:

1) Why should one bother using different diameter rods (or different width rectangular bars) in the filter?

2) Why should one waste the space to put in rods #0 and #$(n+1)$ the only purpose of which is to properly couple the resistive generator and resistive load to the input and output resonances, respectively?

The answer to the first question is apparently that different diameter rods and different width bars are used because the presently available designs call for them. Actually there are an infinite number of correct diameter and spacing combinations, and there is no need to use different diameter rods (or different width bars) if the filter is correctly designed and adjusted for uniform diameter rods (or uniform width bars). Section III gives the simple design procedure involved.

The answer to the second question also is that apparently rods #0 and #$(n+1)$ are used because the presently available designs call for them. Actually in the bandwidth regions considered in this correspondence these rods are not required. In the less than one percent bandwidth region, probe or loop coupling of the generator to resonator #1, and the load to resonator #n is practical; and in the greater than one percent bandwidth region, tapping of the generator onto rod #1, and tapping of the load onto rod #n, is a practical way of producing the required singly loaded Q for the input and output resonators. Section IV gives the simple design procedure involved when tapping is used.

II. THE COEFFICIENT-OF-COUPLING BETWEEN ADJACENT UNIFORM DIAMETER RODS

The definition of coefficient-of-coupling (K) applicable to all small percentage bandwidth coupled-resonator filters is given in Section IV of [2]; also given there is a straightforward experimental procedure for accurately measuring and/or adjusting K. This procedure applied to an actual filter for which $d/h=0.5$ resulted in the three cross-marked points on the graph of Fig. 1. The K measured was that between two adjacent rods which also had equally spaced rods on their other sides; thus, equal far-side coupling was involved.

Within their applicable ranges, either Honey's closed-form equations for Z_{oe} and Z_{oo} [3], or Cristal's graphs for C_m/ϵ and C/ϵ [4] can be used to calculate the coefficient-of-coupling resulting from a given geometry d/h and c/h. It should be noted that the small percentage coupling case herein considered is very forgiving of a multitude of approximation sins, and perhaps a proof of this is the fact that for the less than 10 percent coupling case considered herein, and for normalized rod diameters less than $d/h=0.5$, both procedures give "exactly," i.e., within about 2 per cent, the same numerical answers. If Honey's approximation is used, (1) gives the K between a pair of rods

$$K = \frac{4}{\pi}\left\{\frac{\ln \coth\left(\frac{\pi}{2}\frac{c}{h}\right)}{\ln \coth\left(\frac{\pi}{4}\frac{d}{h}\right)}\right\}. \qquad (1)$$

If Cristal's graphs are used, (2) is used to obtain the K between a pair of rods

$$K = \frac{4}{\pi}\left\{\frac{1}{\dfrac{C_g/\epsilon}{C_m/\epsilon}+2}\right\}. \qquad (2)$$

The graph of Fig. 1, giving the coefficient-of-coupling between two adjacent rods as a function of normalized center to center rod spacing c/h, and normalized rod diameter d/h, results from an application of either (1) or (2). The very excellent agreement on Fig. 1 between the three experimentally determined cross-marked points, and the line for $d/h=0.5$, is gratifying.

As Fig. 1 indicates, in this region of interest, the log of the coefficient-of-coupling is almost a linear function of both the normalized center to center spacing (c/h) and the normalized rod diameter (d/h). A linear equation approximating the straight, almost equally-spaced lines of Fig. 1 is given in (3).

$$\log K \doteq \left\{-1.37\left(\frac{c}{h}\right) + 0.91\left(\frac{d}{h}\right) - 0.048\right\}. \qquad (3)$$

As presented, the equations and graph of Fig. 1 apply exactly only to the case of equal (C_m/C) on each side of every equal diameter rod. In a "modern network-theory filter" this is, of course, not the case; however, for small percentage couplings the foregoing simple equations and graph can still be used to design such an unequal C_m filter, using equal diameter rods, because of the following fundamental fact: Fringing phenomenon is such that, when the spacing between rods is changed, the Y_0 of any one line with all other lines short-circuited, e.g. $(Y_{0_{23}}+Y_{e_3}+Y_{0_{34}})$, changes negligibly; e.g., even with spacing changes such that the resultant coefficient of coupling changes from zero up to 10 percent, there is less than 3 percent change in this Y_0, even for d/h as large as 0.5. The value of this essentially constant Y_0 is very closely that for a single rod between parallel ground planes given by (4) (page 592 of [5]).

$$Y_0 \doteq 1/138 \log\left(\frac{4}{\pi}\frac{h}{d}\right) \qquad (4)$$

Thus, up to 10 percent couplings, the factor $(C_g/\epsilon\ 2C_m/\epsilon)$ in the denominator of (2) is essentially independent of changes in rod spacing; and to a first-order the numerical value for the corresponding numerator of (2), i.e. (C_m/ϵ), is still obtainable from Cristal's Fig. 2 [4], even for unequally spaced rods on each side of a given pair.

It should be noted that a graph similar to Fig. 1 but differing by the factor $4/\pi$, has

Manuscript received May 24, 1965; revised June 2, 1965.